An Adaptive Network of Economic Production Processes

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経済生産過程ネットワークの新しいモデルを説明する。このモデルでは、生産過程は化学反応ネットワークを模して表されている。このシステムでは各生産過程は大域的に結合しており、互いに競合して、その結果外部環境に適応する。このシステムでは、どのパスを用いるかのスイッチが生じ、それを通して経済活動は多様な時間スケールからなるダイナミクスを示す。

1 Introduction and Model

My papers [1] describe our new model of a network of adaptive economic production processes which is a based on a combination of von Neumann's neoclassical model of economic production[2] and the general dynamics of catalytic reaction networks[3].

The original von Neumann model (VNM) of balanced economic growth[2] is described in Fig.1(a). It is considered to be a static equilibrium model which allows von-Neumann to set up and solve some relationships between the variables which must hold at equilibrium. Equilibrium is a state of 'balanced growth' where prices are constant. There are no dynamics defined by the model which might describe out of equilibrium or approach to equilibrium behaviour. It is clear that the original VNM is unrealistic and does not account for many of the problematic, non-equilibrium behaviour of real economies, such as business cycles, recession, unstable prices - inflation and deflation, and unemployment for example.

Our new model[1], is defined by,

$$\frac{dF_i(t)}{dt} = -F_i(t) + \sum_j S_{ij}(t)p_j(t) \tag{1}$$

$$\frac{dS_{ij}(t)}{dt} = -S_{ij}(t) + \frac{F_i(t)\sigma_{ij}(t)}{p_j(t)} + \alpha(b_{ij} - a_{ij})Min_k(\frac{S_{ik}(t)}{a_{ik}}) \tag{2}$$

$$p_j(t) = \frac{\sum_k F_k(t)\sigma_{kj}(t) + D_j^{ext}}{\sum_k S_{kj}(t) + S_j^{ext}} \tag{3}$$

$$\sigma_{ij}(t) = \gamma\frac{p_j(t-\tau)a_{ij}}{\sum_j p_j(t-\tau)a_{ij}} + (1-\gamma)a_{ij} \tag{4}$$

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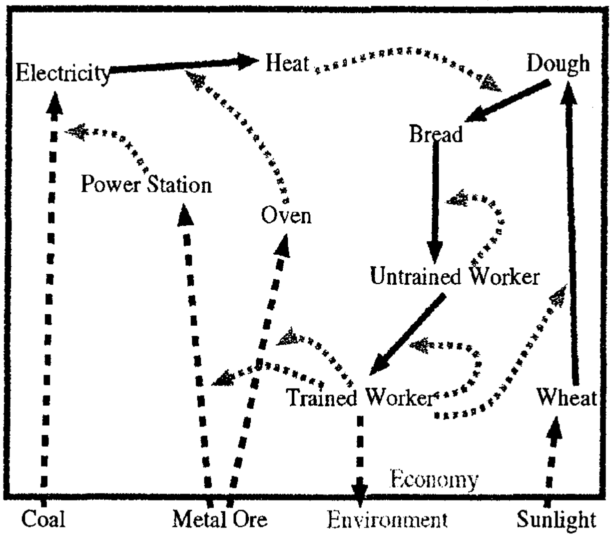

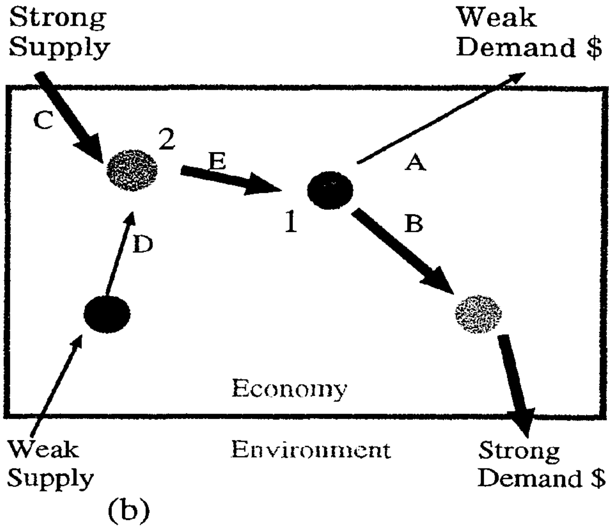

図 1: (a) An VNM Economy describing a fixed set of production processes and products. Solid lines are production flows and dotted lines are catalytic effects. Economic production processes are treated like chemical reactions where capital goods are catalysts. Consumption of goods necessary for the operation of the catalysts is included in process. So for example a process may convert (dough, baker, oven, coal and lunch) into (bread, baker, oven, waste). The stochiometric ratios these products 'react' in are fixed and the baker and oven are catalysts. When many such processes are coupled together we obtain a production economy. Some processes are autocatalytic as shown. The system may be closed or open. In an open system some products are supplied from the environment and others are demanded, shown as long dashed lines. The environment may of course be the set of other economies or other countries. (b) An adaptive network. Circles denote products transformed by production processes which are shown as arrows. In this simple example each process is assumed to only have one input product and one output product and catalysts are not shown for convenience. Product moves from external input supply to external output demand in the positive arrow direction. Funds (i.e. money) move from the external output demand to the external input supply in the negative arrow direction. Arrow size indicates the strength of the process, i.e. its funds size and therefore its share of the available input material. Processes A and B both demand the product 1 and are therefore in competition for it. Process B has access to larger funds than process A so it takes the larger share. Similar processes C and D both supply the product 2 and so are in competion for funds comming from process E. Process C makes product 2 from an external product in larger supply than process D does. Process C then takes a larger share of the available funds and has a larger size. In this way some pathways are *strengthed* while occurs may go *bankrupt* completely. The path from strong supply to strong demand forms a production chain, i.e an assembly line. In this way the network *adapts* or internally evolves to fulfil the *function* of changing its external supplies into external demands in the most efficient way possible.

where i labels the process, and j the product. a_{ij} and b_{ij} are, respectively, input and output stochiometric ratios for process i product j. Each process has supplies $S_{ij}(t)$ of products and also funds $F_i(t)$. S_j^{ext} and D_j^{ext} are external supplies and demands of product j. The model has two parts:

(a) Processing: In our model each process is considered to have supplies of its input products $S_{ij}(t)$. However each process need not possess these supplies in their exact stoichiometric ratios for the production reaction to proceed perfectly without any unused input. To take this into account we have to consider how the rate of the production reaction depends on the supplies.

In a simple chemical reaction system this rate is governed by the law of mass action which states that reactions proceed at rates proportional to the product of the concentrations of the input species. This law which depends on the consideration of *random collisions* as well as on the notion of volume is obviously inapplicable in an economic reaction context. In fact the rate determining quantity in an economic reaction is the quantity of the *minimum* input supply available to the process. For example a baker working at full pace can only fill a certain amount of ovens, increasing the amount of ovens further will not increase the rate of production of bread. And likewise increasing the supply rate of the raw material dough will not increase production if *either* the bakers are already working at full-pace *or* the ovens are full. This processing rate is described by the third term on the RHS of Eq.2.

(b) Marketing. In this part processes continuously exchange goods and funds. Processes continuously send all the supplies and funds to be compared in a central market. This is described by the first term on the RHS of Eq.1,2 where all products and funds are sent to market. In the second terms on the RHS of Eq.1 processes recieve funds $F_i(t)$ in proportion to their supplies $S_{ij}(t)$ and the collectively formed product price $p_j(t)$, Eq.3. In the second terms on the RHS of Eq.2 processes recieve supplies $S_{ij}(t)$ in proportion to their funds $F_i(t)$ and the collectively formed product price $p_j(t)$, Eq.3. The quantities $\sigma_{ij}(t)$ appear because the process must somehow allocate its current funds to the different input products it requires. Eq.4 describes the allocation. When the parameter $\gamma = 0$ the process divides its funds simply by its input stochiometric ratios a_{ij}. When $\gamma > 0$ the process takes information from a previous price $p_j(t-\tau)$ into account. The parameter α in Eq.2 describes the relative timescale of production and marketing.

Unlike the original VNM, because the processes are *globally competitively coupled* through the central market the system becomes an *adaptive network*. See Fig.1(b). This model describes a production economy, or indeed any network of adaptive processes, such as a living cell as a network of molecular machines[4], much more realistically than the VNM and sheds light on the microscopic origin of non-equilibrium economic phenomena such as described above.

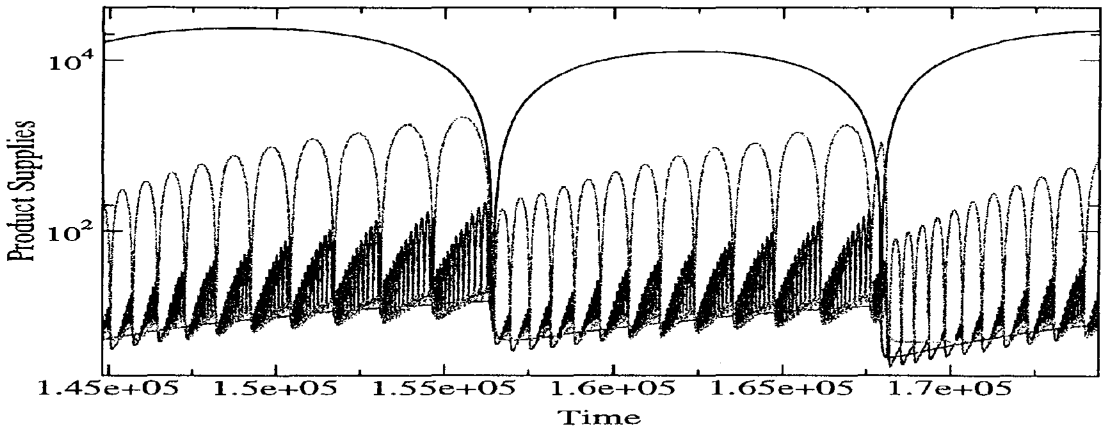

図 2: Time series of *Hierarchical multiple timescale limit cycle attractor* in process chain in fixed environment. There are 3 processes and 7 products. The processes, denoted by (input, catalyst, output) are (1,2,3),(3,4,5),(5,6,7). 3 products are catalysts (2,4,6), one in each process each with a fixed external supply S^e_{iC} and demand D^e_{iC}. There is one input product I (1) with a fixed external supply S^e_I and one output product O (7) with fixed external demand D^e_O. There are two intermediate products (3,5) with no external supplies and demands. Here price feedback $\gamma = 0.9$.

2 Results and Discussion

A characteristic of this model is the emergence of multiple hierarchical timescales dynamics, caused by the minimum condition threshold production kinetics[1]. According to this minimum condition the network dynamically switches between different production pathways. Mulitple timescale dynamics are illustrated in the economically relevant production chain shown in Fig.2. This may be the basic reason for multiple timescale business cycles and non-equilibrium behaviour in econcomics in general, and further investigation is in progress.

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